Exercise 1. For each of the following complex numbers, determine its real part and its imaginary part.

$$z_1 = -3 - 2i, \qquad z_2 = 25, \qquad z_3 = -i + 2,$$

$$z_4 = \frac{i}{2}, \qquad z_5 = -(2+i) + (-5i+1), \qquad z_6 = \frac{\sqrt{2}}{2}i,$$

- $z_1 = -3 - 2i \qquad \mathrm{Re}(z_1) = -2 \;;\; \mathrm{Im}(z_1) = -3$
- $z_2 = 25 \qquad \mathrm{Re}(z_2) = 25 \;;\; \mathrm{Im}(z_2) = 0$
- $z_3 = 2 - i \qquad \mathrm{Re}(z_3) = 2 \;;\; \mathrm{Im}(z_3) = -1$
- $z_4 = \frac{i}{2} \qquad \mathrm{Re}(z_4) = 0 \;;\; \mathrm{Im}(z_4) = \frac{1}{2}$
- $z_5 = -(2+i) + (-5i+1)$

  $= -2 - i - 5i + 1$

  $= -1 - 6i \qquad \mathrm{Re}(z_5) = -1 \;;\; \mathrm{Im}(z_5) = -6$
- $z_6 = \frac{\sqrt{2}i}{2} \qquad \mathrm{Re}(z_6) = 0 \;;\; \mathrm{Im}(z_6) = \frac{\sqrt{2}}{2}$

Exercise 2. Write each of the following complex numbers in algebraic form.

$$z_1 = i^2, \quad z_2 = (2+i)^2, \quad z_3 = i^3, \quad z_4 = (1+i)(1-i).$$

$z_1 = i^2$

$= -1$

$z_2 = (2+i)^2$

$= 4 + 2i - 1$

$= 3 + 2i$

$z_3 = i^3$

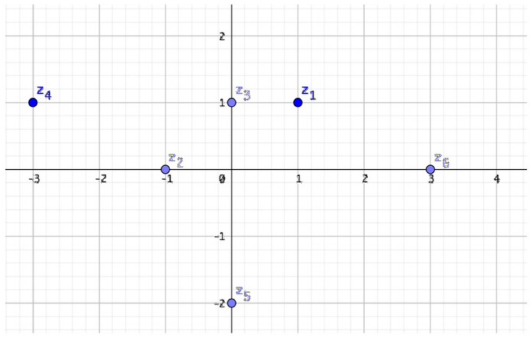
$= -i$

$z_4 = (1+i)(1-i)$

$= 1 - i + i + 1$

$= 2$

Exercise 3. Write each of the complex numbers represented below in the complex plane in algebraic form.



Read from graph:

$z_1 = 1 + i$ | $z_3 = i$ | $z_5 = -2i$

$z_2 = -1$ | $z_4 = -3 + i$ | $z_6 = 3$

Exercise 4. Given $z = 2 + i$ and $w = 1 + 3i$ compute:

$$-w, \quad \frac{1}{w}, \quad \frac{\bar{z}}{w}.$$

$-w = -1 - 3i$

$$\frac{1}{w} = \frac{1}{1+3i} \cdot \frac{1-3i}{1-3i}$$

$$= \frac{1-3i}{10}$$

$$= \frac{1}{10} - \frac{3}{10} i$$

$$\frac{\bar{z}}{w} = \frac{\overline{2+i}}{1+3i} \cdot \frac{1-3i}{1-3i}$$

$$= \frac{(2-i)(1-3i)}{10}$$

$$= -\frac{1}{10} - \frac{7}{10} i$$

Exercise 5. Write the following complex numbers in algebraic form:

$$z_1 = \frac{1}{i}, \quad z_2 = \frac{1+i}{2-i}, \quad z_3 = \frac{-2+i}{-1+i}.$$

$$z_1 = \frac{1}{i} \cdot \frac{i}{i}$$

$$= -i$$

$$z_2 = \frac{1+i}{2-i}$$

$$= \frac{(1+i)(2+i)}{5}$$

$$= \frac{1}{5} + \frac{3}{5} i$$

$$z_3 = \frac{(-2+i)(-1-i)}{(-1+i)(-1-i)}$$

$$= \frac{2+2i-i+1}{1+i-i+1}$$

$$= \frac{3}{2} + \frac{1}{2} i$$

Exercise 6. Using the following formula:

$$|a+ib| = \sqrt{a^2+b^2},$$

compute the modulus of the following numbers:

$$z_1 = i, \qquad z_2 = 1+i, \qquad z_3 = -2+3i, \qquad z_4 = \frac{1}{i}, \ .$$

$|z_1| = 1$

$|z_2| = \sqrt{1^2+1^2}$
$= \sqrt{2}$

$|z_3| = \sqrt{2^2+3^2}$
$= \sqrt{13}$

$z_4 = -i$

$|z_4| = 1$

Exercise 7 (From the algebraic form to the trigonometric form)**.** Consider the complex number:

$$z = 1+i.$$

1. Write the trigonometric form of z.

2. Let z' be a complex number with modulus $|z'| = 1$ and argument $\arg(z') = \frac{\pi}{3}$. Write z' in trigonometric and exponential form.

3. Write z' in algebraic form.

1. $z = 1+i$

$|z| = \sqrt{1^2+1^2}$
$= \sqrt{2}$

$\cos(\theta) = \frac{a}{r}$
$= \frac{1}{\sqrt{2}}$
$= \frac{\sqrt{2}}{2}$

$\sin(\theta) = \frac{b}{r}$
$= \frac{\sqrt{2}}{2}$
$\Rightarrow \theta = \frac{\pi}{4} + 2k\pi, \ k \in \mathbb{Z}$

Therefore: $z = 1+i$
$= \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$
$= \sqrt{2}\,\mathrm{cis}\left(\frac{\pi}{4}\right)$

2. Trigonometric form: $z' = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$
$= e^{i\frac{\pi}{3}}$

3. $\theta = \frac{\pi}{3} \Rightarrow \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ et $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore : $z' = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

Exercise 8. Express the following complex numbers in exponential form.

$$z_1 = \sqrt{3} + i, \qquad z_2 = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}, \qquad z_3 = -1,$$

$|z_1| = \sqrt{\sqrt{3}^2 + 1} = 2$ | $\cos(\theta) = \frac{a}{r} = \frac{\sqrt{3}}{2}$ | $\sin(\theta) = \frac{b}{r} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6} + 2k\pi$ with $k \in \mathbb{Z}$

Therefore: $z_1 = 2e^{i\frac{\pi}{6}}$

$|z_2| = \sqrt{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$

$$\left.\begin{aligned} \cos(\theta) &= \frac{a}{r} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{\sqrt{2}}{2} \\ \sin(\theta) &= \frac{b}{r} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{\sqrt{2}}{2} \end{aligned}\right\} \Rightarrow \theta = \frac{\pi}{4} \mod (2\pi)$$

Therefore: $z_2 = \frac{1}{2} e^{i\frac{\pi}{4}}$

$z_3 = -1$ Euler's identity: $e^{i\pi} + 1 = 0$

$\Leftrightarrow e^{i\pi} = -1$